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MHD FREE CONVECTION FLOW THROUGH A POROUS MEDIUM BOUNDED BY AN INFINITE VERTICAL PLATE WITH CONSTANT HEAT FLUX

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ABSTRACT

This paper considers the problem of free convective flow of a viscous incompressible fluid through a porous medium bounded by an infinite vertical porous plate in the presence of transverse magnetic field. The plate is subjected to a normal suction velocity and the heat flux at the plate is constant. The governing partial differential equations are transformed into ordinary differential equations by means of similarity transformations. Numerical solution of the resulting coupled ordinary differential equations using Runge-Kutta-Fehlberg Forth-Fifth order method is obtained. The effect of various parameters are presented and discussed.

KEYWORDS : Free convection flow, Transverse magnetic field, Constant heat flux, Numerical study

NOMENCLATURE

B_0	Constant applied magnetic field
C_p	Specific heat at constant pressure
g	Gravity acceleration
Gr	Grashof number
K	Permeability parameter
M	Magnetic parameter
Nu	Nusselt number
Pr	Prandtl number
$\overline{q}$	Dimensional heat flux from the plate
T	Temperature of the fluid
u,v	Velocity component of the fluid along the x and y directions, respectively
x,y	Cartesian coordinates along the surface and normal to it, respectively

GREEK SYMBOLS

β	Thermal expansion coefficient
ρ	Density of the fluid
σ	Electrical conductivity
μ	Viscosity of the fluid
κ	Thermal conductivity
υ	Kinematic viscosity
θ	Dimensionless temperature

SUPERSCRIPT

$-$	Dimensional quantities
'	Derivative with respect to

SUBSCRIPTS

w	Properties at the plate
∞	Free stream condition

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The study of flows through porous media has been motivated by its immense importance and continuing interest in many engineering and technological field, for example, soil mechanics, petroleum engineering, transpiration cooling, food preservation, cosmetic industry blood flow and artificial dialysis etc. Free convective phenomenon has been the object of extensive research, because it is often encountered in cooling of nuclear reactor or in the study of the structure of stars and planets etc, from the technological point of view, MHD free convection flows have also great significance for the application in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics, (Cramer and Pai ; 1977).

The theory, of laminar flow through, homogeneous porous media is based on an experiment originally conducted by Darcy (1856). The free convection flow past a vertical plate studied by Kolar et al., (1988) and Ramanajah et al., (1992) with different boundary conditions. Problem of natural convective cooling of a vertical plate solved numerically by Camargo et at., (1996) Raptis (1986) studied mathematically the case of time varying two dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. Helmy (1998) analyzed MHD unsteady free convection flow past a vertical porous plate embedded in a porous medium. Elabashbeshy (1997) studied heat and mass transfer along a vertical plate in the presence of magnetic field. Chemkha and Khaled (2001) investigated the problem of coupled heat and mass transfer by magnetohydrodynamic free convection from an inclined plate in the presence of internal heat generation or absorption. Dursunkaya and Worek (1992) studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafonssian and Willams (1995) presented the same effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Alam and Rahmam (2005) studied the Dufour and Soret effect on study MHD free convective heat and mass transfer flow past a semi infinite vertical porous plate embedded in a porous medium. Anavda et al. (2009) studied thermal diffusion and chemical effects with simultaneous thermal and mass diffusion in MHD mixed convection flow. Jhankal and Kumar (2014) studied MHD plane poiseuille flow with variable viscosity and unequal wall temperature. Chen (2004) has studied MHD free convection from an inclined surface with suction effects. Anwar Beg and Ghosh (2010) studied the steady and unsteady MHD free and forced convective flow of electrically conducting, new thorium fluid in the presence of appreciable thermal radiation heat transfer and surface temperature oscillation.

Motivated by works mentioned above and practical applications, the main concern of the present paper is to study the effects of a magnetic field on the free convective flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux.

MATHEMATICAL FORMULATION

Consider the axis of be taken along the vertical plate in the upward direction and axis perpendicular to it, the applied magnetic field is of uniform strength and is applied transversely to temperature differences except that the density in the body force term. The momentum with temperature is negligible. The porous material containing the fluid is, in fact, a non-homogeneous medium, but it is possible to replace it with a homogeneous fluid that has dynamical properties equal to the local averages of the original non-homogeneous continuum. Thus, the complicated problem of the motion of a viscous fluid in a porous solid reduces to the motion of the homogeneous fluid with some reduces to the motion of homogeneous fluid with some additional resistance.

We consider the case of short circuit problem in which the applied electric field E=0, and also assure that the induced magnetic field is small compared to external magnetic field this implies a small magnetic Reynolds number. Under these assumption the equation, which govern the motion are (Raptis, 1983).

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad \text{...(1)}$$

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = g\beta(\bar{T}-\bar{T}_\infty) + \bar{v}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\nu}{\bar{K}}\bar{u} - \frac{\sigma B_0^2}{\rho}\bar{u} \qquad \text{...(2)}$$

$$\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\kappa}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\nu}{C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \qquad \text{...(3)}$$

Where all the symbols have their usual meanings. The last two terms on the right hand side of equation (2) signify the additional resistance due to the porous medium, with permeability $\bar{K}$, and the electromagnetic body force term which acts on the fluid elements, respectively. Also, the Joule heating terms in the energy equation (3) is assumed to be negligible. The boundary conditions are:

$$\bar{y} = 0: \bar{u} = 0, \frac{\partial \bar{T}}{\partial \bar{y}} = -\frac{\bar{q}}{\kappa}$$

$$\bar{y} \to \infty: \bar{u} \to 0, \bar{T} \to \bar{T}_\infty \qquad \text{...(4)}$$

The continuity equation (1) gives

$$\bar{v} = -v_0 \qquad \text{...(5)}$$

Where $v_0 > 0$ is the constant suction velocity at the plate and the negative sign indicates that the suction velocity is directed towards the plate.

Introduce the following non-dimensional quantities:

$u = \frac{\bar{u}}{v_0}$ (Velocity), $y = \frac{v_0 \bar{y}}{\nu}$, $\theta = \frac{\bar{T}-\bar{T}_\infty}{\bar{q}\nu/\kappa v_0}$ (Temperature),

$Pr = \frac{\rho\nu C_p}{\kappa}$ (Prandtl number),

$Ec = \frac{\kappa v_0^3}{\bar{q}\nu C_p}$ (Eckert number), $Gr = \frac{g\beta\nu^2\bar{q}}{\kappa v_0^4}$(Grashof number),

$K = \frac{v_0^2\bar{K}}{\nu^2}$(Permeability parameter), ...(6)

$M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}$ (Magnetic field parameter).

On putting these values into equations (2) and (3), we get

$$u''(y) + u'(y) = -Gr\theta(y) + \frac{u(y)}{K} + Mu(y) \qquad \text{...(7)}$$

$$\theta''(y) + Pr\theta'(y) = -PrEcu'^2(y) \qquad \text{...(8)}$$

Along, with the boundary conditions:

$$y = 0: u = 0, \theta' = -1$$

$$y \to \infty: u \to 0 \;\; \theta \to 0 \qquad \text{...(9)}$$

NUMERICAL SOLUTIONS

The non- liner coupled differential equations (7) and (8) subject to boundary condition (9) are solve numerically using Runge-Kutta-Fehlberg forth-fifth order method. To solve these equations we adopted symbolic algebra software Maple. Maple uses the well known Runge-Kutta-Fehlberg Fourth-fifth order (RFK45) method to generate the numerical solution of a boundary value problem. The boundary condition were replaced by those at y=7 in accordance with standard practice in the boundary layer analysis. The effect of various parameters on velocity distribution and rate of heat transfer in terms of Nusselt number are shown in figure 1 and 2 respectively.

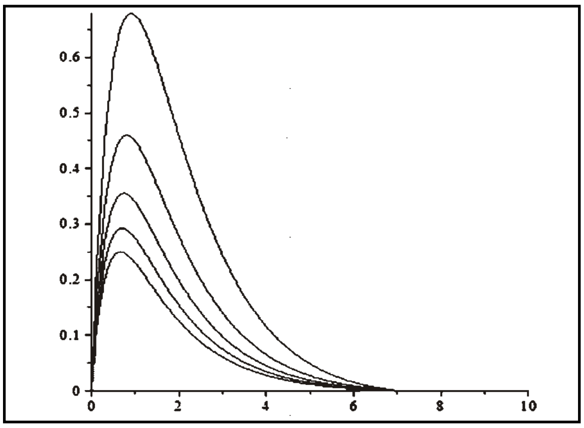

Figure 1: Variation of velocity for various values of magnetic parameter M, when Pr=0.71, Gr=2.0, K=1.0 and Ec=0.01

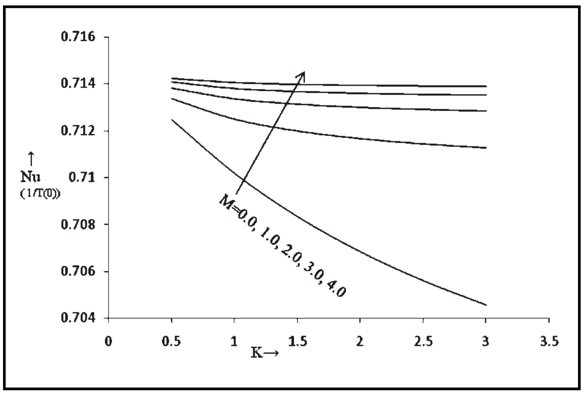

Figure 2: Variation of Nusselt number against permeability parameter K for various values of magnetic parameter M, when Pr=0.71, Gr=2.0 and Ec=0.01

CONCLUSION

A mathematical model has been presented for the effects of a magnetic field on the free convective flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux. From the study, following conclusions can be drawn:

(i) Figure 1 shows the Maple generated numerical solutions for a Prandtl number 0.71, Grashof number 2.0, Eckert number 0.01, permeability parameter 1.0 and for a range of values of the magnetic parameter (M). We notice that the velocity decreases as the magnetic parameter (M) increases. Thus we conclude that we can control the velocity field by introducing magnetic field.

(ii) Figure 2 shows the rate of heat transfer in terms of Nusselt number with the permeability parameter K, for a Prandtl number 0.71, Grashof number 2.0, Eckert number 0.01 and for a range of values of the magnetic parameter (M). We notice that the rate of heat transfer is increases as the magnetic parameter (M) increases, this heat transfer is very important in production engineering to improve the quality of the final product.

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